A LESSON ABOUT TESSELLATIONS, FOR THE CLASSROOM OR A CLUB

The artist who failed maths ended up inspiring mathematicians

A 50–60 minute lesson for GCSE Foundation and Higher, and NCEA Level 1. It also works with curious students in Years 7–10, with an extension for those ready for more. It pairs with the short film *Maths in the News #3* and the student worksheet. The aim is a clear geometric reason, not exam technique.

What you will need

- The student worksheet (pages 1–4; keep page 5, the answers, for yourself)
- The Short, if it is available — or the spoken hook below
- Spare paper (or printed squares) and scissors for the cut-and-slide activity — please do not have students cut the worksheet itself
- Coloured pencils, optional
- Optional: the lesson page at ilovemaths.app/lessons/escher-tessellations

Objectives

- Know that angles at a point add to 360°, and use this to decide whether a shape will tessellate.
- Calculate the interior angle of a regular polygon, and show why only equilateral triangles, squares and regular hexagons tessellate by themselves.
- Identify a translation, a rotation and a reflection.
- Make a cut-and-slide tile and explain why it still tiles, with the same area as the square.

Curriculum

GCSE Foundation and Higher (angles at a point; interior angles of regular polygons; tessellation as an application; translations, rotations and reflections). NCEA Level 1 (geometric reasoning with angle properties; transformations). The extension, on semi-regular tessellations, is a natural step for Higher or a fast group.

The shape of the lesson

0–8 min — **Starter hook — play the Short if you can.** The film is 84 seconds long. If it is not yet on YouTube, use the spoken hook: “A museum in New York is celebrating the artist M.C. Escher, until 8 February 2027. As a student he struggled with maths, and later said he ‘never got a pass mark.’ His tilings still inspire mathematicians. Before we look at why some shapes tile a floor and others don’t: can you tile a bathroom floor with regular pentagons, with no gaps?” Take two or three answers. Do not correct yet. Ask what questions the film (or the hook) left them with.

8–24 min — **Main — angles, then which polygons tessellate.** Pages 1–2 of the worksheet. Read the 360° idea together, then let students complete the interior-angle table (Q3). Circulate. If someone uses the exterior angle by mistake, ask “is that the angle inside the polygon, or the turn at a corner?” Then Q4–Q9, with the diagrams. The pentagon’s 36° gap is the “aha” moment — linger there. For the octagon (Q8), 135 × 2 leaves a gap and 135 × 3 overlaps; do not rush it.

24–48 min — **Activity — transformations, then make a tile.** Page 3 for Q10–Q13, then page 4 to design. Keep the transformation questions short (a few minutes). For the cut-and-slide, demonstrate once on the board or with paper: cut a piece from one side of a square, slide it to the opposite side, and show two copies fitting together. Then let students invent their own motif on spare paper. If you have only 50 minutes, keep the making to about 15 minutes and leave the extension for homework.

48–55 min — **Plenary.** In one sentence each, orally or on a mini whiteboard: “Regular pentagons leave a gap because…” and “The cut-and-slide tile still works because…”. Listen for 108° × 3 = 324°, a 36° gap, and “the bump fills the hole.” Thank effort and good questions by name, specifically. If there is time, hold up two or three of their tiles.

Discussion questions you can drop in

- Why 360°? What else in maths uses a full turn?
- If regular pentagons leave a gap, does that mean *no* pentagon can tile a floor? (Some irregular pentagons can — a nice aside, not required today.)
- Why might an artist who struggled with school maths still do deep mathematics in pictures?
- Is a tessellation the same as "a pattern that repeats"? What would you have to add to that description?

Misconceptions to listen for

- **Any regular polygon will tessellate if you use enough of them.** Q9 is written for this. The interior angle must divide 360°, and at least three polygons must meet at a point.
- **Mixing up interior and exterior angles.** The exterior angle of a regular n-gon is $360° / n$. That is a useful fact, but it is not the angle that has to fit around a point when you tile.
- **Thinking six hexagons must meet at a point,** by analogy with six triangles. A regular hexagon is 120°, so only three fit.
- **Thinking the cut-and-slide changes the area,** or that you may rotate the cut piece. The slide is a translation: no turning. The area is unchanged because the piece is only moved.
- **Confusing translation, rotation and reflection.** Keep the three L-shape diagrams in view while they answer Q10 and Q11.
- **Over-claiming about pentagons.** Regular pentagons do not tessellate. Some irregular pentagons do. If a student raises this, praise it and park it for the extension note at the foot of page 4.

Differentiation

If students need more support

Give the interior-angle formula, and complete the triangle and the square as a class before pairs finish the table. For Q7, calculate 108 × 3 together and let students only subtract from 360. For the tile, offer a pre-drawn zigzag to slide, or a printed square with a cut line already marked. Pair work is better than a solo struggle with scissors.

If students are ready for more

The extension on page 4 (E1–E3) is for Higher, NCEA Level 1 students who want a stretch, or a fast Foundation group. E1 (the 4.8.8 arrangement) is a gentle start. E2 asks them to invent another mix. E3 is the richest: a table for $n = 3$ to 8 is enough, and a factor argument ($n - 2$ divides 4) is a nice bonus. A further challenge: cut and slide on *both* pairs of sides of a square.

The November 2026 news — please keep this careful

The hook is a current exhibition, *Mathematics and the Art of M.C. Escher*, at the National Museum of Mathematics (MoMath) in New York, until 8 February 2027 (MoMath's page gives 1 October 2026 through 8 February 2027). Quanta Magazine published "How M.C. Escher's Art Creates Bizarre Mathematical Worlds" by Shalma Wegsman on 18 November 2026. It reports that Escher struggled with mathematics as a student, once remarking that he "never got a pass mark," and that the insights in his artwork became an enduring inspiration to mathematicians. Please use that short quoted phrase, attributed to Escher, as reported by Quanta. This plan did not check the original book in which a longer form of the remark is often cited, so stay with the short, widely reported wording. The teenage-friendly title line is "The artist who failed maths ended up inspiring mathematicians"; the precise wording of the quote is "never got a pass mark." Do not tell students that Escher invented the classroom cut-and-slide construction, and please do not use or photocopy Escher's prints for this lesson — the diagrams are original on purpose.

Links

- This lesson on I Love Maths: **https://ilovemaths.app/lessons/escher-tessellations**
- The Short, once it is published, and Jane's channel: youtube.com/@getsussed-y4h
- The student tutor app (enjoyment and confidence, not exam marks): https://ilovemaths.app
- MoMath, the exhibition: https://momath.org/escher/
- Shalma Wegsman, Quanta Magazine, 18 November 2026: https://www.quantamagazine.org/how-m-c-eschers-art-creates-bizarre-mathematical-worlds-20261118/

For teachers: how students learn best with AI

The evidence is still early, and it is mixed. These points are here to be useful and honest — not to oversell I Love Maths, or AI in general. Links were checked in November 2026.

1. **AI is more likely to help when it supports understanding than when it does the work.** A November 2026 review by Stanford's SCALE Initiative (the AI Hub for Education) looked at 52 cause-and-effect studies. It found that using AI to understand new material is more promising than using it to produce the work — although neither use consistently improves learning. Using AI to offload the task can reduce what students later know.
2. **Treat sweeping "AI improves learning" claims with care.** An Education Week report (2 November 2026) covered an audit of 14 research reviews that all claimed positive effects. The audit found they failed basic research standards: they mixed very different studies, and often lumped together engagement, motivation or "how much students felt they had learned" with actual achievement.
3. **A few careful trials are encouraging, but they are still small.** In one randomised trial in Ghana, students who used Rising Academies' WhatsApp maths tutor Rori for two 30-minute sessions a week, over eight months, gained about 0.37 standard deviations in maths compared with a control group (around 1,000 students in grades 3–9, across 11 schools). That is a meaningful gain in education research, and the authors asked that these first-year results be interpreted carefully.
4. **Good design coaches step by step, rather than handing over the answer.** Google's Guided Learning mode in Gemini is built to ask questions and walk through the steps, rather than simply giving the answer. That is a design choice, in line with the idea above. It is not, by itself, independent proof that the tool improves results.
5. **When AI is used to mark work, human checks still matter.** NZQA uses automated text scoring for digitally submitted NCEA Literacy writing, and human markers check scripts around the Achieved / Not Achieved boundary (about 40% of assessments, in NZQA's published description). If the human mark differs, the human mark is used. This is about assessment marking, not tutoring — it is a reminder that AI works best as a support, with people still in the loop.
6. **The older findings about how people learn still apply, with or without AI.** Students generally learn more when they have to pull ideas out of memory (retrieval practice), explain their thinking (self-explanation), work through a genuine challenge rather than being rescued too quickly (productive struggle), and get feedback on their method, not only on the final answer.

How I Love Maths uses this

I Love Maths is built to support understanding, not to do the work for the student. The tutor asks questions before it gives answers. To reach "Mastered" on a topic, a student needs to type an explanation in their own words. If they send a photo of their working, it is marked for method — and the photo is not stored.

Sources (checked November 2026):

1. Stanford SCALE, "The Evidence Base on AI in K-12: A November 2026 Review" — https://scale.stanford.edu/ai/evidence-base-review/evidence-base-ai-k12-november-review
2. Stephen Sawchuk, "The Research on AI in Education Is a Mess", *Education Week*, 2 November 2026 — https://www.edweek.org/technology/the-research-on-ai-in-education-is-a-mess/2026/11
3. Owen Henkel, Hannah Horne-Robinson, Nessie Kozhakhmetova and Amanda Lee, "Effective and Scalable Math Support: Evidence on the Impact of an AI-Tutor on Math Achievement in Ghana" (arXiv, May 2024 version) — https://arxiv.org/abs/2402.09809
4. Google, "How Gemini's Guided Learning can help you study more effectively", 23 September 2025 — https://blog.google/products-and-platforms/products/gemini/guided-learning-google-gemini/
5. NZQA, "How we mark co-requisite assessments" — https://www2.nzqa.govt.nz/ncea/subjects/litnum/marking/
6. Retrieval practice: https://www.retrievalpractice.org/ · Michelene Chi and colleagues, "Eliciting Self-Explanations Improves Understanding", *Cognitive Science*, 1994 — https://onlinelibrary.wiley.com/doi/10.1207/s15516709cog1803_3 · Manu Kapur, "Productive Failure in Learning Math", *Cognitive Science*, 2014 — https://onlinelibrary.wiley.com/doi/full/10.1111/cogs.12107 · John Hattie and Helen Timperley, "The Power of Feedback", *Review of Educational Research*, 2007 — https://journals.sagepub.com/doi/10.3102/003465430298487

Jane, an experienced maths teacher
I Love Maths — a friendly maths tutor for GCSE and NCEA students who want maths to make sense again.
https://ilovemaths.app · This lesson: https://ilovemaths.app/lessons/escher-tessellations