A LESSON ABOUT SINGULARITIES AND FLOW

Are there singularities in your coffee?

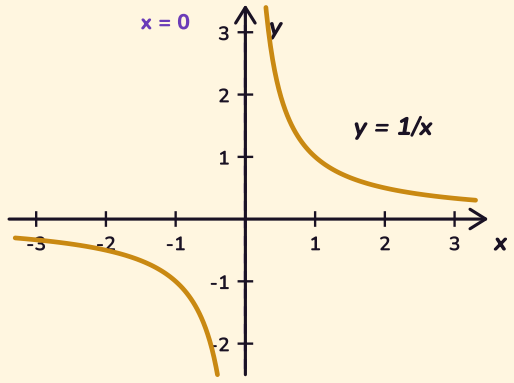
This week a podcast said there are singularities in your coffee. In maths, the word means something much more precise — and more interesting. Today you will see what "grows without bound" looks like on a graph, how speed is a rate, and how to check a news claim. The film's caveat applies all the way through: **ideal fluid, not your coffee**.

Name _____ Class / year _____ Date _____

1. What does "grows without bound" mean?

A quantity **grows without bound** if it becomes larger than any number you can name, if you go far enough — or close enough to a point. On a graph, the curve shoots up (or down) towards a vertical dashed line. That line is a **vertical asymptote**: the graph approaches it but does not cross it.

In this story, a **singularity** is where a description breaks down: some quantity grows without bound, or a solution stops being smooth. It is not a black hole, and it is not the "AI singularity" from the podcasts.



As x gets close to 0 from the right, y grows without bound

Q1. Complete the table for $y = 1/x$.

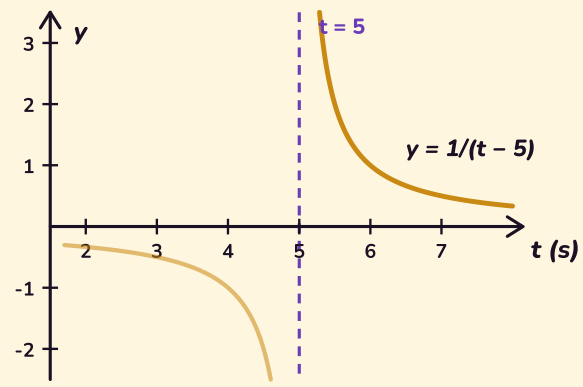
x	Working	y = 1/x
1		
0.5		
0.2		
0.1		
0.01		
0.001		

Q2. As x gets closer to 0 from the positive side, what happens to y?

Q3. The graph has a vertical asymptote. Where is it, and what does that dashed line mean?

2. A blow-up at a finite time

Now let t be time in seconds, and take $T = 5$. Consider $y = 1/(t - 5)$. This is the same kind of graph as $y = 1/x$, but the vertical asymptote is at a time you can point to on a clock.



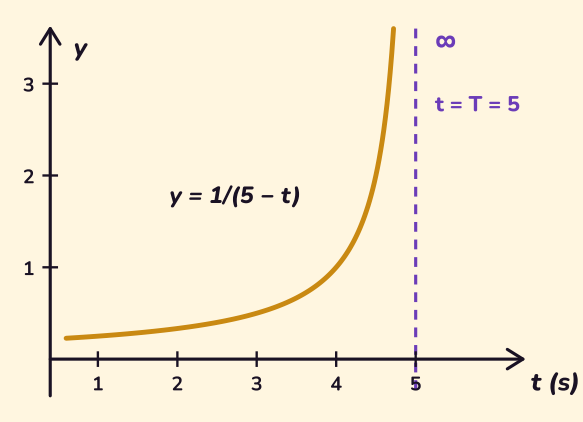
As t gets close to 5 from the right, y grows without bound

Q4. Complete the table for $y = 1/(t - 5)$.

t (seconds)	Working	y
6		
5.5		
5.2		
5.1		
5.01		
5.001		

Q5. As t gets closer to 5 from the right, what happens to y? Does this happen at a time you can name (a **finite time**)?

Mathematicians often write $1/(T - t)$ for a quantity that grows as time runs towards T from below. With $T = 5$, that is $y = 1/(5 - t)$. It is a mirror of the graph above, and it stays positive as t approaches 5.



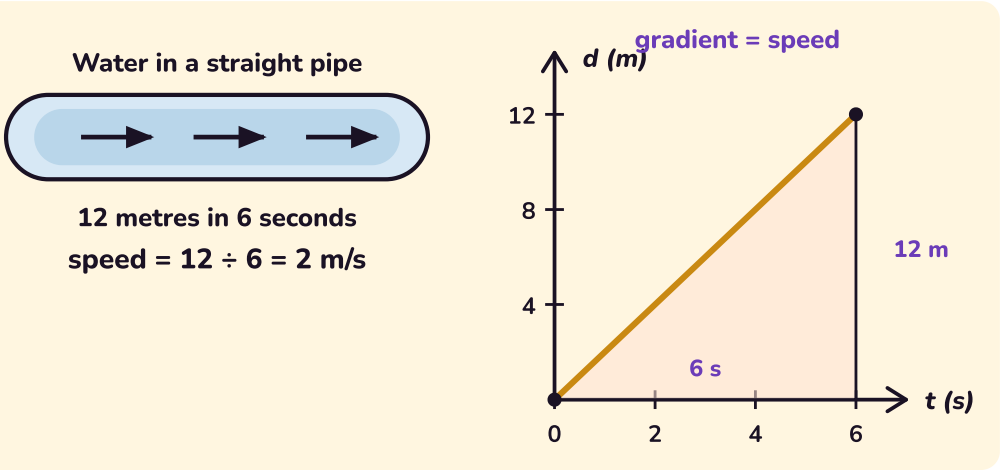
A blow-up at a finite time: y shoots up as t approaches 5

Q6. Complete this shorter table for $y = 1/(5 - t)$, as time runs towards 5.

t (seconds)	y = 1/(5 − t)
4	
4.5	
4.8	
4.9	
4.99	

3. Rates of change and flow

Speed is a rate: how quickly distance changes. On a distance–time graph, the **gradient** is the speed. The Navier–Stokes equations model how fluids such as water and air flow. The million-dollar question, in plain words, is: if the flow starts smooth, can the speed become infinite in a finite time?



Q7. Water travels 12 metres along a straight pipe in 6 seconds. What is its average speed? Show your working, including the units.

Q8. The distance–time graph is a straight line from (0, 0) to (6, 12). What is the gradient of that line, and what does the gradient represent?

Q9. A float on a river drifts 10 metres in 4 seconds.

(a) What is its average speed?

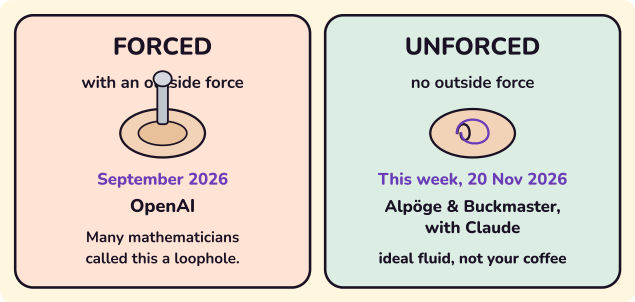
(b) If that speed stayed constant, how far would it travel in 20 seconds? Is this the same kind of question as “can the speed become infinite”?

Q10. In one sentence, what would a “yes” to the Clay question look like on a speed–time graph?

> **A careful caveat.** Everyday water in a pipe or a river does not do this. The prize question is about a carefully built **ideal** mathematical fluid — not your morning coffee, and not the river in Q9.

4. Forced or unforced?

Since 2000, a Clay $1 million prize has asked whether a smooth Navier–Stokes flow can blow up with **no extra stirring**. In September 2026, OpenAI announced a proof of blow-up, but only after adding a specially designed outside force. This week, Alpöge and Buckmaster, working with Claude, claimed a blow-up with no force at all — the real prize question, if it holds.



Q11. In your own words, what is the difference between a **forced** claim and an **unforced** claim?

Q12. Why does the film say “ideal fluid, not your coffee”?

5. Evidence-checking card-sort

For each claim, write one verdict: **Credible**, **Unclear**, **Misleading** or **False**, and a short reason. You may cut the claims into cards if you prefer to sort them on the desk.

	Claim	Verdict	One-line reason
C1	The Navier–Stokes equations model the flow of fluids such as water and air.		
C2	In this story, a singularity means a black hole in your coffee.		
C3	Since 2000, a Clay Millennium Prize problem has offered $1 million for an answer to this question: if a Navier–Stokes flow starts smooth, can the speed become infinite in a finite time?		
C4	OpenAI solved the Clay prize problem in September 2026, and the $1 million has been awarded.		
C5	In September 2026, OpenAI announced a proof that a smooth 3D flow can blow up in finite time, but only with a carefully constructed outside force. Many mathematicians called this a loophole.		
C6	Clay said the problem had “apparently been settled”, so Navier–Stokes is now officially solved.		
C7	Alpöge and Buckmaster, working with Claude, have settled the unforced prize question: the proof has been independently confirmed.		
C8	The Clay Institute still lists the problem as open, the key evidence for this week’s claim has not been released, and a fair verdict today is: unclear.		

6. Extension

E1. What is the same about the graphs of $y = 1/(t - 5)$ and $y = 1/(5 - t)$? What is different?

E2. A quantity starts at 1 and doubles every second. After 10 seconds it is 1,024. After 20 seconds it is 1,048,576. Does this become infinite in a finite time? How is that different from $1/(5 - t)$?

E3. Clay’s prize rules require publication in a qualifying outlet, at least two years, and general acceptance. Why might a committee wait, even after a dramatic headline?

Answers — for teachers, or for checking afterwards

Q1. The values of y are **1, 2, 5, 10, 100, 1,000**. Working: 1÷1, 1÷0.5, 1÷0.2, 1÷0.1, 1÷0.01, 1÷0.001.

Q2. y becomes larger and larger — it grows without bound. Students may say it "heads towards infinity" or "gets bigger than any number you name." Accept any clear version.

Q3. The vertical asymptote is the line $x = 0$ (the y-axis). The graph approaches that dashed line but does not cross it.

Q4. The values of y are **1, 2, 5, 10, 100, 1,000**. Working: 1÷(6−5), 1÷0.5, 1÷0.2, 1÷0.1, 1÷0.01, 1÷0.001. It is the same arithmetic as Q1, with $t - 5$ in place of x.

Q5. y grows without bound. Yes: it happens as t approaches 5 seconds, which is a finite time you can point to on a clock.

Q6. The values of y are **1, 2, 5, 10, 100**.

Q7. Average speed = 12 ÷ 6 = **2 m/s** (or 2 metres per second). Units matter.

Q8. Gradient = rise/run = 12÷6 = **2**. The gradient represents the speed, 2 m/s.

Q9. (a) 10 ÷ 4 = **2.5 m/s**. (b) 2.5 × 20 = **50 m**. No: this is ordinary finite speed, staying constant. It is not a speed that grows without bound.

Q10. Look for the idea of a speed–time graph shooting up towards a vertical asymptote at some finite time T. Students do not need formal wording.

Q11. A forced claim adds an outside force (like a hand still stirring the cup). An unforced claim is about the fluid on its own, with no extra stirring — the question the prize is actually about.

Q12. "Singularities in your coffee" is a vivid metaphor, not a statement about a real cup. The claimed blow-up is for a specially constructed ideal incompressible fluid, not everyday stirring.

C1. Credible. This is what the Navier–Stokes equations are for (water and air are the usual examples).

C2. False. In this story a singularity is where a description breaks down — a quantity growing without bound, or a solution stopping being smooth. It is not a black hole, and it is not about a real cup of coffee.

C3. Credible. Clay named existence and smoothness of 3D Navier–Stokes solutions one of seven Millennium Prize Problems in 2000, with a $1 million prize. In plain words, the question is whether a smooth flow can develop infinite speed in a finite time.

C4. False. OpenAI announced a proof only for the forced case, and said it does not intend to claim the prize. Clay still lists the problem as Active. The prize has not been awarded.

C5. Credible. That is a fair summary of OpenAI's 8 September 2026 announcement and of what Scientific American reported on 21 September 2026.

C6. Misleading. Clay did say the problem had "apparently been settled" and that its evaluation is "deliberately unhurried." It still lists Navier–Stokes as Active, not solved. Stopping at "settled" overstates the official position.

C7. False. They claimed a finite-time blow-up with no external force, written with help from Claude, on 20 November 2026. The key evidence (a certificate) had not been released as of 25 November 2026, and there was no independent confirmation. The problem is not settled.

C8. Credible. Clay still lists the problem as open; the key evidence has not been released; nobody has independently confirmed the proof. The film's verdict is **unclear**.

E1. Both have a vertical asymptote at $t = 5$, and both grow without bound as t approaches 5. They are mirror images: $1/(5 - t)$ is positive as t approaches 5 from below; $1/(t - 5)$ is positive as t approaches 5 from above (and negative from below).

E2. After 20 seconds the quantity is huge but still finite (2^{20} = 1,048,576). It only tends to infinity as the number of seconds tends to infinity — not at a time you can name. By contrast, $1/(5 - t)$ grows without bound as t approaches the finite time 5. That is the difference between "eventually enormous" and "a blow-up in finite time."

E3. Clay's rules ask for publication, at least two years, and general acceptance. Checking a claim — especially when the key evidence has not been released — takes time. A news headline is not the same as awarding the prize.

Diagrams on this worksheet are original. Sources for the news claims are listed on the lesson page and in the teacher lesson plan.